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Local Equilibrium Assumption for Round Jet Calculations

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Introduction

A FAMILY of similarity solutions has been found[1] for self-preserving, incompressible, turbulent round jets. The similarity parameter is $\eta = r/\delta(x)$, where x and r are the axial and radial coordinates, respectively, $\delta(x)$ the jet half-width, defined as the width of the jet at $U/U_0(x) = 0.5$, U is the mean velocity of the jet and U_0 is the jet centerline velocity. All memebers of the family lead to power-law decay for δ and $U_0(x)$, namely $\delta \alpha x$ and $U_0 \alpha x^{-1}$; however, they differ in the behavior of the eddy viscosity, ν_t. The classical Tollmien[2] solution is given by constant ν_t. Other similarity solutions are also possible depending on the variation of $\nu_t(\eta)$. Some of these solutions are not physically valid, even though they are similarity solutions to the jet governing equations, because they lead to zero ν_t at the jet centerline. One member among those that give rise to finite $\nu_t(0)$ is found to give better agreement with the mean and turbulent flux measurements of Wygnanski and Fiedler[3] and Chevray and Tutu[4] than the classical Tollmien[2] solution. Since ν_t decreases from $\nu_t(0)$ to zero as $\eta \to \infty$, it is more realistic than constant ν_t and is in better quantitative agreement with measured ν_t.

The purpose of this Note is to make use of these similarity solutions to obtain closed-form similarity solutions to the k and ϵ equations and, in the process, assess the influence of the local equilibrium assumption on round jet calculations. These closed-form solutions will be compared with the numerical similarity solution obtained by Vollmers and Rotta[5] using a k-kl closure model and the k-ϵ model calculation of Launder et al.[6] It will be shown that while the neglect of the equilibrium assumption leads to a decay of the centerline turbulent kinetic energy, k_0, as x^{-2}, the asumption gives a decay of x^{-1}. The latter result seems to be in better quantitative agreement with measurements.

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Analysis

Transport of turbulent kinetic energy k and the dissipation rate of k, ϵ, in an incompressible round jet is governed by the equations[6]

$$U\frac{\partial k}{\partial x} + V\frac{\partial k}{\partial r} = \frac{1}{r}\frac{\partial}{\partial r}\left(\frac{r\nu_t}{\sigma_k}\frac{\partial k}{\partial r}\right) + \nu_t\left(\frac{\partial u}{\partial r}\right)^2 - \epsilon \tag{1}$$

$$U\frac{\partial \epsilon}{\partial x} + V\frac{\partial \epsilon}{\partial r} = \frac{1}{r}\frac{\partial}{\partial r}\left(\frac{r\nu_t}{\sigma_\epsilon}\frac{\partial \epsilon}{\partial r}\right) + C_{\epsilon 1}\frac{\epsilon}{k}\nu_t\left(\frac{\partial U}{\partial r}\right)^2 - C_{\epsilon 2}\frac{\epsilon^2}{k} \tag{2}$$

where U and V are the mean velocities along x and r, respectively, σ_k and σ_ϵ are the Prandtl numbers for k and ϵ, and $C_{\epsilon 1}$ and $C_{\epsilon 2}$ are model constants. The values assumed by Launder et al.[6] for σ_k, σ_ϵ, $C_{\epsilon 1}$, and $C_{\epsilon 2}$ are 1, 1.3, 1.9, and 1.4, respectively. The boundary conditions for k and ϵ are $k(x,0) = k_0(x)$, $k(x,\infty) = 0$, $\epsilon(x,0) = \epsilon_0(x)$, and $\epsilon(x,\infty) = 0$, where $k_0(x)$ and $\epsilon_0(x)$ are centerline values of k and ϵ and have to be determined as part of the solutions. If the local equilibrium assumption or production of k balances the dissipation of k is made; Eqs. (1) and (2) are then reduced to forms similar to the mean momentum equation for round jets. This suggests that Eqs. (1) and (2) can be analyzed in the manner suggested by So and Hwang.[1] Here, only the most valid solution is used to analyze Eqs. (1) and (2). According to Ref. 1, that solution is given by

$$U = U_0 e^{-\eta^2 \ln 2} \tag{3}$$

$$V = \frac{4U_0(\ln 2)^2}{Re_t}\left[\eta e^{-\eta^2 \ln 2} + \frac{e^{-\eta^2 \ln 2} - 1}{2\eta \ln 2}\right] \tag{4}$$

$$\frac{\nu_t}{\nu_t(0)} = \frac{1 - e^{-\eta^2 \ln 2}}{\eta^2 \ln 2} \tag{5}$$

$$\delta' = \frac{4(\ln 2)^2}{Re_t} \tag{6}$$

$$\frac{U_0'}{U_0} = -\frac{\delta'}{\delta} \tag{7}$$

where $Re_t = [U_0\delta/\nu_t(0)]\ln 2$, the turbulent Reynolds number, is taken to be constant and is determined from jet spread, the primes denote differentiation with respect to x. Substituting Eqs. (3-7) into Eqs. (1) and (2), invoking the assumption $\nu_t(\partial U/\partial r)^2 = \epsilon$, and making use of the boundary conditions, the following solutions for k and ϵ are obtained:

$$\frac{k}{k_0} = \frac{\epsilon}{\epsilon_0} = e^{-\eta^2 \sigma_k \ln 2} \tag{8}$$

$$\frac{k_0'}{k_0} = -\frac{\delta'}{\delta} \tag{9}$$

$$\frac{\delta\epsilon_0'}{\epsilon_0} = -\frac{4(\ln 2)^2\sigma_k}{\sigma_\epsilon Re_t}(1 + \sigma_k - \sigma_\epsilon) \tag{10}$$

$$\frac{1}{\sigma_\epsilon} = \frac{1}{\sigma_k} + \frac{Re_t(C_{\epsilon 2} - C_{\epsilon 1})}{4(\ln 2)^2\sigma_k^2}\frac{\delta\epsilon_0}{k_0 U_0} \tag{11}$$

With the results, $\delta \alpha x$ and $U_0 \alpha x^{-1}$ from Eqs. (6) and (7), the decay of k_0 and ϵ_0 can be deduced from Eqs. (9) and (10) to be $k_0 \alpha x^{-1}$ and $\epsilon_0 \alpha x^{-3}$. Equation (11) gives a relation for the determination of σ_ϵ once Re_t, σ_k, $C_{\epsilon 1}$, and $C_{\epsilon 2}$ are specified. On the other hand, a similarity solution for k (even though

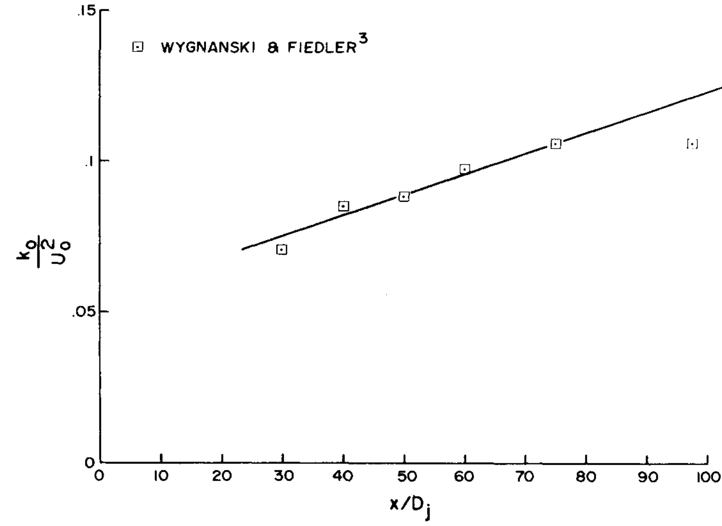

Fig. 1 Decay of centerline turbulent kinetic energy.

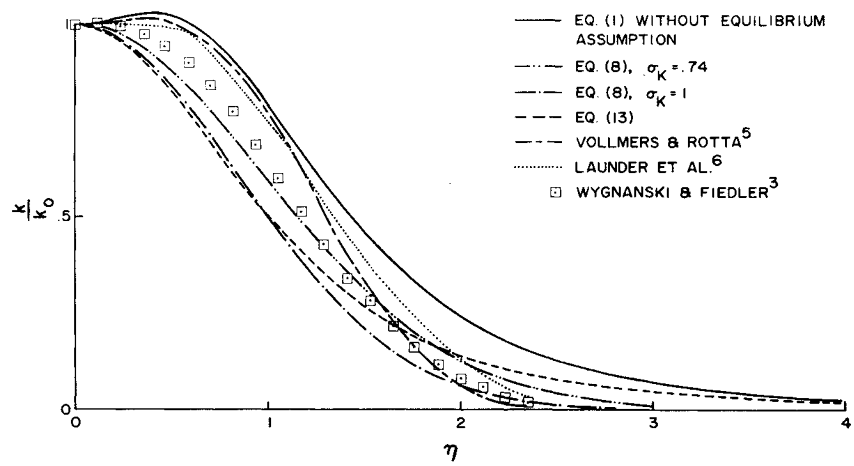

Fig. 2 Comparison of the k distributions for round jet.

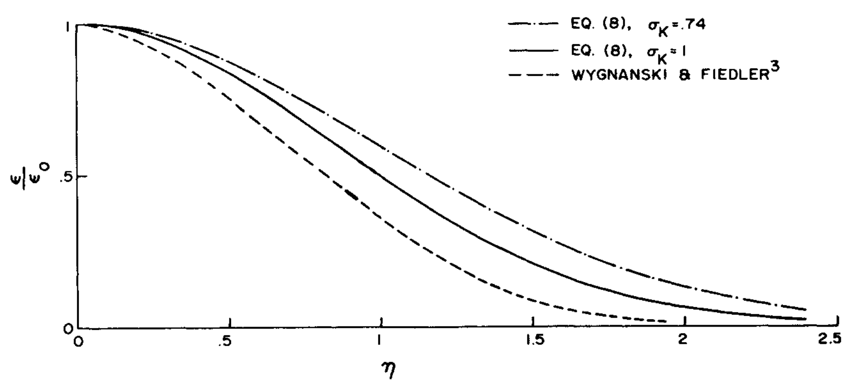

Fig. 3 Comparison of the isotropic dissipation rate ϵ for round jet.

not in closed form) can be found for Eq. (1) without invoking the local equilibrium solution.[5] However, the condition for similarity requires that

$$k_0/U_0^2 = \text{const} \tag{12}$$

which means that $k_0 \alpha x^{-2}$. If constant ν_t is assumed instead of Eq. (5), then Tollmien's[2] solution for U and V would give rise to

$$\frac{k}{k_0} = \frac{1}{[1+(\sqrt{2}-1)\eta^2]^2} \tag{13}$$

for Eq. (1) under the local equilibrium assumption.

Discussion

From the measurements of Wygnanski and Fiedler,[3] the behavior of k_0/U_0^2 vs x/D_j, where D_j is the jet diameter, is shown in Fig. 1. With the exception of the data point at $x/D_j = 97.5$, the measurements can be essentially correlated by a straight line, thus suggesting that self-preserving jet flow should be analyzed by invoking the local equilibrium assumption.

A comparison of the k distributions determined from Eq. (8) with $\sigma_k = 0.74$ and 1, and from Eq. (13) with the measurements of Wygnanski and Fiedler[3] is shown in Fig. 2. Also shown is the result of solving the self-similar form of Eq. (1) without invoking the local equilibrium assumption and with U/U_0 and V/U_0 given by Tollmien's results,[2] the numerical similarity solution of Vollmers and Rotta,[5] and the solution of Launder et al.[6] using a k-ϵ model of turbulence. It can be seen that the best correlation with measurements is given by Eq. (8) with $\sigma_k = 0.74$, and the worst agreement by the self-similar solution of Eq. (1) without assuming local equilibrium of turbulence. This clearly indicates the influence of the local equilibrium assumption on the k distribution, and points to the fact that such an assumption is quite valid for self-preserving round jets.

Consistent with the local equilibrium assumption, one would expect the turbulence field to be isotropic. This suggests a comparison of Eq. (8) with isotropic dissipation rate data.[3] The comparison (see Fig. 3) is reasonably good qualitatively. However, as expected, Eq. (8) is not in agreement with the semi-isotropic measurements.[3] Because there are no data on ϵ_0, the decay law for ϵ_0 cannot be verified. In spite of this, an indirect verification of $\epsilon_0 \alpha x^{-3}$ can be obtained by considering the expression $\nu_t = C_\mu k^2/\epsilon$ used in the k-ϵ model of turbulence. Launder et al.[6] find it necessary to decrease C_μ with x, otherwise the jet spread will be overpredicted. If $k_0 \alpha x^{-1}$ and $\epsilon_0 \alpha x^{-3}$ are assumed, then $\nu_t = \nu_t(\eta)$ only if $C_\mu \alpha x^{-1}$, which is in qualitative agreement with the suggestion of Launder et al.[6]

Conclusion

It may be said that $\nu_t(\eta)$ prescribed by Eq. (5) gives the best correlation with measurement for k and further substantiates the claim of So and Hwang[1] that the solution [Eqs. (3-7)] represents the best solution for self-preserving, turbulent round jets. Furthermore, Eqs. (3-7) also represent the solutions for incompressible heated round jets. If the turbulent Prandtl number Pr_t is assumed to be constant, then Eq. (3) with the exponent $\eta^2 \ln 2$ replaced by $\eta^2 Pr_t \ln 2$ becomes the solution of the temperature equation. Since Eqs. (1) and (2) are equally applicable for incompressible heated round jets, Eqs. (8-11) also represent solutions for such flows. Therefore, the present results are just as valid for nonisothermal round jets.

Acknowledgment

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Numerical Solution to Rarefaction or Shock Wave/Duct Area-Change Interaction

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Introduction

THERE is a growing interest in the simulation of the flowfields arising from the interaction of rarefaction or shock waves with a segment of the area change in a duct of an elsewhere uniform cross section. We believe that this class of flow problems constitutes a useful model for the analysis of the phenomena encountered in various areas of engineering practice and research, such as piping systems of reciprocating pumps or engines, gas transportation pipelines, shock tubes, and blast wave simulators that incorporate area-change segments in their driver or channel sections.

Early attempts at solving such flowfields[1,2] were hampered by the lack of adequate computers. Only flows in uniform cross sections were computed as truly nonstationary, using the method of characteristics. The flow across each area change was treated as being steady.[1,2] With the advent of modern computers, fully nonstationary solutions became feasible. One of the frequently used numerical schemes is the random choice method (RCM).[3-5] In RCM, which is of first-order accuracy, shock waves and contract discontinuities are sharply defined, unlike schemes employing artificial viscosity (explicit or implicit in the scheme) that generally "smear" discontinuous jumps over several mesh points.

Recently, the diffraction of shock waves or rarefaction waves from an area change in a duct were studied by Greatrix and Gottlieb,[6] Gottlieb and Igra,[7] and Irgra and Gottlieb[8] using the RCM. The results of these computations can exhibit excessive noise in the computed spatial distributions of pressure and velocity that seems to be directly related to the way in which the flowfield in an element containing a shock wave is chosen, with a probability proportional to the length of pre- or aft-shock portion within the element. This noisy distribution was particularly strong in a large area ratio case (e.g., see Figs. 22 and C5 in Ref. 6, Fig. 9 in Ref. 7, and Fig. 9 in Ref. 9). Igra et al.[9] were able to reduce the noise by using finer grid and a more suitable random number generator. However, they were not able to eliminate it altogether (see Figs. 22-25 of Ref. 9).

The purpose of this Note is to demonstrate the advantage gained by using the higher-order numerical scheme based on a generalized Riemann problem (GRP).[10-13] Its running cost was much lower than that of the fine-grid RCM computations, since noise-free and high-resolution results were obtained with far fewer grid points. (Due to the numerical stability limitation on the integration time step, the amount of computation is proporational to the number of grid points squared in both the RCM and GRP schemes.)

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